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Performance-sharing optimization by risk-constrained equity investors

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ABSTRACT

An investment project may be capital constrained when its risk exceeds the risk limit of prospective investors. We propose a new equity-contract in which the project's performance-sharing across investors respects the individual investor's risk limit while staying as close as possible to his/her percentage contribution in equity of the project. The proposed arrangement of performance-sharing thus ensures that the investors with constrained risk limits take less share of performance during high-risk episodes, while the less constrained investors are more exposed. The former pay a premium to the latter to compensate for the partial risk transfer. The proposed performance-sharing agreement is expected to be especially useful for risk-constrained equity investors who are restricted in their use of risk-free investments to reduce investment risk.

1. Introduction

Investors set their investment objectives under constraints on the risk of their periodic returns. Often, this implies setting a risk limit that their periodic returns will not fall below a threshold under a certain probability level (Das et al., 2010). These constrained investors may be reluctant to invest in a project with positive net present values but substantial risks in future values. A conventional solution for the problem is to offset the excess of risk with risk-free assets. This solution is not permissible for Islamic investors (Arslan-Ayaydin et al., 2018). It may not be feasible for conventional investors when they face full investment constraint in risky assets or not desirable in periods of negative interest rates on risk free investment(Corneille et al., 2020).

We propose to account for the risk limits of such investors in the definition of a performance-sharing rule that can deviate from the traditional capital allocation-based one. Through a proposed agreement between investors, we transform single project returns into distinct investor returns constrained by their risk limits. The transformation involves two steps. First, at the start of each period, the investors agree on a performance share that is the closest to their capital contribution and constrained by their risk limits. Second, at the end of the period, the performance is distributed according to the ex ante agreed performance shares unless the losses are so extreme that one party needs to contribute additional capital. In that case, there is an ex post truncation, which we detail in the paper. We refer to this proposed sharing rule as performance risk-limits matching (PRISMA).

This system of performance-sharing is especially beneficial for the most risk-constrained investor. We recommend a compensation to the counterparty taking more risk. This happens through the payment of a premium. To balance the risk-return rearrangement

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between the investors, we calibrate premium values at break-even level in their expected risk-adjusted returns. This arrangement may allow us to fix the premium without violating no-arbitrage and gharar (Shari'a) restrictions. In fact, the ex ante agreement to transfer future risk is also possible under Shari'a rules if we adopt *takaful al ta'awuni* (El-Gamal, 2006) conditions.[1]

We organize the remaining parts of the paper in three sections. In the next section we introduce the proposed PRISMA model with required variable and optimization settings. In Section 3, we apply the model as an agreement between two risk limit types with single investor in each category and present our analysis of the results. In the last section, we conclude with final remarks.

2. Criteria for performance-sharing optimization

We begin the section with general investment settings and define its transformation into PRISMA performance-sharing rule. After these general settings, we define the ex ante optimization for performance-sharing ratios under the proposed system along with ex post truncation and premium calibration.

2.1. General settings

Assume at time t, N investing agents start an investment project, with each agent $i \in N$ contributing $V_{i,t}$ amount, such that the project initial value is $V_t = \sum_{i=1}^{N} V_{i,t}$ and the investment contribution ratio of agent i is $w_{i,t} = \frac{V_{i,t}}{V_t}$. In conventional finance, the future returns of the project $r_{t+1} = \frac{V_{t+1} - V_t}{V_t}$, at time $t+1$, are distributed among investors as follows:

$$r_{i,t+1} = \frac{V_{i,t+1} - V_{i,t}}{V_{i,t}} = \frac{w_{i,t} V_t r_{t+1}}{w_{i,t} V_t} = r_{t+1}. \tag{1}$$

The performance-sharing mechanism in Eq. (1) shares the project performance equally and does not recognize the fact that the investors may be risk-constrained. We propose an alternate mechanism of performance-sharing that accounts the risk limits of the investors.

2.2. Proposed performance-sharing rule

We introduce a performance risk-limit matched (PRISMA) performance-sharing rule that requires an agreement between investors to share future performance of a risky project in a ratio that is based on ex ante risk and constrained by their risk limits. We propose that at each point in time t and for an ex ante risk $\mathcal{R}_{t+1|t}$ of the project, each agent i with risk limit λ_i agree with the other agents to share the future performance of the project at time $t+1$ in a ratio $e_{i,t}$ that ensures

$$e_{i,t} \mathcal{R}_{t+1|t} \leq \lambda_i w_{i,t}, \tag{2}$$

where $\mathcal{R}_{t+1|t}$ is the predicted risk for the project return over the horizon from t to $t+1$, conditional on the information available at time t. The performance-sharing constraint in Eq. (2) guarantees that during high-risk conditions, no risk-constrained investor is allocated share of expected risk $e_{i,t}\mathcal{R}_{t+1|t}$ more than their risk acceptance threshold defined in terms of their invested capital $\lambda_i w_{i,\ t}$. The ex ante sharing ratio $e_{i,t}$ is then applied at time $t+1$, which transforms the project returns into distinct investor returns,

$$r_{i,t+1} = \frac{V_{i,t+1} - V_{i,t}}{V_{i,t}} = \frac{e_{i,t} V_t r_{t+1}}{w_{i,t} V_t} = \frac{e_{i,t}}{w_{i,t}} r_{t+1}. \tag{3}$$

Without loss of generality, we group investors into two distinct risk limit categories, particularly the low risk limit (LRL) and the high risk limit (HRL) investors. Suppose that the risk limits can be defined as λ_l and λ_h for LRL and HRL investors, respectively, such that $\lambda_l \leq \mathcal{R}_{t+1|t} \leq \lambda_h$.

The performance-sharing under the proposed performance-sharing mechanism in Eqs. (2) and (3) is especially beneficial for the LRL investors, as it protects them during high-risk conditions from risk allocation over and above their risk thresholds. To compensate the HRL investors for taking more risk, we recommend charging a periodic premium π from the LRL investors:

$$V_{i,t}^{\pi} = V_{i,t} - \pi 1_{\lambda_i = \lambda_l} V_{i,t} + \pi 1_{\lambda_i = \lambda_h} V_{i,t}. \tag{4}$$

This compensation setting transforms the investor returns in Eq. (3):

$$r_{i,t+1}^{\pi} = \frac{V_{i,t+1}^{\pi} - V_{i,t}^{\pi}}{V_{i,t}^{\pi}} = \frac{e_{i,t} V_t r_{t+1}}{w_{i,t}^{\pi} V_t} = \frac{e_{i,t}}{w_{i,t}^{\pi}} r_{t+1}, \tag{5}$$

where

[1] To define premium in Shari'a terms, we consider premium as *tabarru/hibah mashrutah bi'iwad*, whose value may be fixed and is legally enforceable to demand contractual indemnity under the agreement (Archer et al., 2011). The application of *tabarru* and the premium is the same, in terms of calculations

$$w_{i,t}^{\pi} = V_{i,t}^{\pi} \Big/ V_t. \tag{6}$$

The performance-sharing ratios $e_{i,t}$, under PRISMA performance-sharing rule, are proposed to be as close as possible to the premium-adjusted investor contribution $w_{i,t}^{\pi}$ to capital of the project and constrained by investor risk-acceptance threshold λ_i.

2.3. PRISMA-PSR ex ante optimality

For an ex ante project risk $\mathcal{R}_{t+1|t}$ at time t, and for the investor risk limit λ_i and the premium-adjusted capital contribution $w_{i,t}^{\pi}$, the optimal share of investor performance $e_{i,t}$ can be defined by a solution to the following quadratic program:

$$\begin{aligned} &\text{minimize}_{e_t \in \mathcal{C}} \sum_{i=1}^{N} \left(w_{i,t}^{\pi} - e_{i,t} \right)^2 \\ &\text{subject to} \begin{cases} 0 \le e_{i,t} \le 1 \\ \sum_{i=1}^{N} e_{i,t} = 1 \\ e_{i,t} \le \lambda_i \dfrac{w_{i,t}^{\pi}}{\mathcal{R}_{t+1|t}}, \end{cases} \end{aligned} \tag{7}$$

where $\mathcal{C}$ is the feasible set. The solution to the optimization problem in Eq. (7) minimizes the divergence of new performance-sharing ratios $e_{i,t}$ from premium-adjusted capital invested ratio $w_{i,t}^{\pi}$. The system also reduces the chances of loss allocation to the investors, higher than their risk limit.

It is important to understand that the optimal solution for proposed performance-sharing $e_{i,t}$ in Eq. (7) is based on ex ante risk measure and is applied on ex post project returns in Eq. (5). In extreme risk conditions, this performance-sharing may lead to an ex post share of losses that exceed the invested capital of HRL investors. To avoid this undesirable feature, we add an ex post adjustment ensuring that no investor is allocated more losses than their invested capital, while remaining as close as possible to the ex ante solution.

2.4. PRISMA-PSR ex post adjustment

For period t, ex ante performance-sharing ratios $e_{i,t}$, and the ex post losses r_t, the adjusted performance-sharing ratios $\rho_{i,t}$ are defined by solution to the following quadratic program:

$$\begin{aligned} &\text{minimize}_{\rho_t \in \mathcal{D}} \sum_{i=1}^{N} \left(e_{i,t-1} - \rho_{i,t} \right)^2 \\ &\text{subject to} \begin{cases} 0 < \rho_{i,t} < 1 \\ \sum_{i=1}^{N} \rho_{i,t} = 1 \\ \dfrac{\rho_{i,t}}{w_{i,t-1}^{\pi}} r_t \ge -1. \end{cases} \end{aligned} \tag{8}$$

where $\mathcal{D}$ is the feasible set. The model reallocates actual losses such that all losses are accounted for and investors are allocated losses not more than their capital.

2.5. Premium calibration

Under the performance-sharing agreement, the LRL investors reduce their exposure to risk at the detriment of the HRL investors. The HRL investor thus requires a premium to compensate for the higher risk exposure, while the LRL investor is willing to pay for this. A challenging question is to determine the equilibrium premium. In this paper, we take a practical approach and set the premium such that the expected risk-adjusted returns are equal. The risk-adjusted returns equal the average return divided by their volatility, as defined in Boudt et al. (2019). Their expected counterpart is computed by resampling techniques. We use a grid search to set the premium to the smallest value satisfying all risk limit constraints while achieving the break-even condition in terms of expected risk-adjusted returns.

1. GJR Model Specifications:
   (a) Let t be daily time intervals, then the location scale for daily log returns is:

   $$R_t = \mu_t + \sigma_t \epsilon_t,$$

   where R_t is the daily log-return with conditional mean μ_t and standard deviation σ_t, and ϵ_t the standardized error term with zero mean and unit volatility;
   (b) The variance equation under the GJR-GARCH model is:

   $$\sigma_{t+1}^2 = \omega + (\alpha + \gamma \mathbb{1}_{R_t < \mu_t})(R_t - \mu_t)^2 + \beta \sigma_t^2,$$

   where the coefficients ω, α, and β are all positive;
   (c) Parameter Estimation: We estimate daily parameters on a rolling basis from inception using Gaussian quasi-maximum likelihood.
2. Nonparametric Monthly VaR:
   (a) Let m represent the number of days in the risk evaluation period;
   (b) Let μ_m and $\overline{\sigma}_m$ be predicted mean and volatility for the m days returns;
   (c) Let $\overline{r}_m$ be the centered returns in the sample around μ_m then;

   $$\overline{r}_m = r_m - \mu_m.$$

   (d) Let $\overline{z}_m$ be m days standardized return and calculated as

   $$\overline{z}_m = \overline{r}_m / \overline{\sigma}_m.$$

   (e) Let $\alpha = 5\%$ and $z_m(\alpha)$ be the α historical quantile of $\overline{z}_m$, then we set:

   $$VaR_m^{\alpha} = \mu_m + z_m(\alpha)\sigma_m.$$

Algorithm 1. Monthly (m days) semiparametric VaR.

Table 1
Out-of-sample results for PRISMA contract applied to US stock returns.

Scenarios	Risk limits	Ann. ret.	Ann. std. dev	Ann. RAR	Av.($\rho_{i,t}$)	Av.($w^{\pi}_{i,t}$)
RLS1	$\lambda_l = 0.03$	0.068	0.085	0.794	0.201	0.269
	$\lambda_h = 1$	0.130	0.163	0.794	0.799	0.731
RLS2	$\lambda_l = 0.09$	0.085	0.132	0.645	0.441	0.446
	$\lambda_h = 1$	0.096	0.149	0.645	0.559	0.554
RLS3	$\lambda_l = 0.15$	0.088	0.138	0.637	0.480	0.481
	$\lambda_h = 1$	0.091	0.143	0.637	0.520	0.519
Conventional		0.0883	0.1411	0.626	0.5	0.5

Note: The table shows out-of-sample results for three risk-limit scenarios for both LRL and HRL investors. We show annualized returns, standard deviations, and risk-adjusted returns under PRISMA performance-sharing rule for each investor in each scenario against conventional project results in the last row. We also show the PRISMA-adjusted average weights of invested capital and the ex post performance-sharing ratios for each investor under each scenario.

3. Illustration for PRISMA performance-sharing rule

In this section, we illustrate PRISMA performance-sharing rule for out-of-sample analysis on aggregate US stock returns over the period from January 1980 to October 2019.[2]

3.1. Calibration

As a risk measure, we use the value at risk (VaR) because it is considered to be the industry standard for downside risk. We assume the PRISMA-rule to be set each month (22 working days) and the monthly VaR be calculated through semiparametric quantile method at the 5% probability level. The procedure to calculate the monthly VaR is described in Algorithm 1, which uses a burn-in sample of 2000 days. The out-of-sample evaluation period is thus from January 1988 to October 2019.

We assume two investor types with *low and high risk limits*, with single risk limit in each category, such that the performance of the project under PRISMA performance-sharing rule is shared between them. This agreement may include single or multiple investors on each side of the risk limit type. In case of more than one investor in any investor type, the investor in each category will share the eventual performance in proportional capital contribution of such category. We set three different risk limit scenarios: RLS1 ($\lambda_l = 0.03$, $\lambda_h = 1$), RLS2 ($\lambda_l = 0.09, \lambda_h = 1$), and RLS3 ($\lambda_l = 0.15, \lambda_h = 1$).

We use grid search method to find minimum value for premium adjustment that guarantees equal expected risk-adjusted returns for both investor types on post-PRISMA results. The calibrated premium at each risk limit scenario are 0.37% for RLS1, 0.1245% for RLS2, and 0.0395% for RLS3.

3.2. Results for proposed performance-sharing rule

The application of PRISMA performance-sharing rule transforms investor returns, performance share $\rho_{i,t}$ and capital allocation weights $w^{\pi}_{i,t}$. The results in comparison to conventional sharing mechanism are reported in Table 1 and Fig. 1.

The results in Table 1 show that PRISMA performance-sharing rule guarantees equal expected risk-adjusted returns for both LRL and HRL investors. The LRL investor has lower returns with lower volatility, and the HRL investor has higher returns with higher volatility. PRISMA performance-sharing rule also guarantees that the LRL investor's risk sharing ratios ($\rho_{i,t}$) reduce with decreasing risk-limits.

Fig. 1 shows the corresponding cumulative investment values. We see that for RLS1, where the LRL investor has a very low monthly risk-limit (0.03), there is the highest gap between their cumulative invested values. This is partly because of the higher premium charged from the LRL investor. The gap between cumulative investment values of the investors and the project is reducing as we increase the monthly risk-limit of the LRL investor.

4. Conclusion

A risky investment project may lose prospective investment from risk-constrained investors when its time-varying risks exceed their risk-limits. We introduce an alternate mechanism to make performance-sharing, flexible enough to adapt to the investor risk-limits.

Our proposed model successfully allocates the project performance in accordance with the investor risk limits, even at extreme risk limits. The model allows for LRL investors, to limit their risks and keep their over-all returns competitive in comparison to the conventional risk-free securities.

[2] We take S&P 500 adjusted price index as a proxy for the project value evolution. While this index is not a representative for a typical investment project, we choose it for illustration as it is available for a long time span.

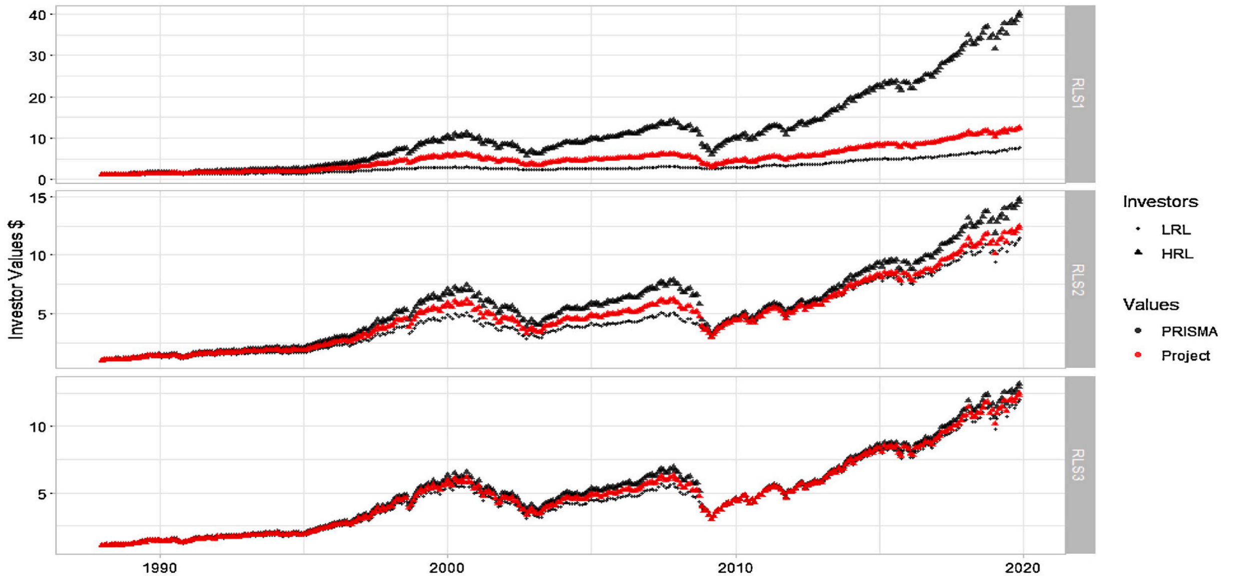

Fig. 1. Cumulative investment value under PRISMA performance-sharing rule *Note:* The grids from top to bottom show investor values under PRISMA performance sharing rule, for three risk limit scenarios (RLS), namely, RLS1, RLS2 and RLS3.

The project risk and the fair premium value may be difficult to estimate for risky projects. This creates model risk in the proposed performance sharing rearrangement. The model risk is higher, when there is high uncertainty about the project cashflows, when the total project duration is longer, or when the risk prediction horizon is longer.

In this research paper, we have taken only two investors with very low and very high risk limit and created a one-to-one contractual mapping for premium-indemnity mechanism. In the future, we would like to propose a generalized mechanism where in each account we can take more than one risk-limit.

CRediT authorship contribution statement

Kris Boudt: Conceptualization, Methodology, Writing - review & editing, Supervision. **Mulazim-Ali Khokhar:** Data curation, Software, Investigation, Visualization, Methodology, Validation, Writing - original draft, Writing - review & editing.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.frl.2020.101527

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